**PROBABILITY AND RANDOM VARIABLES**

**Random Experiment:** The experiments which are performed essentially under the same conditions and the results cannot be predicted are known as random experiments.

**Ex:** Tossing a coin, rolling a die etc.

**Sample space:** The set of all possible outcomes of a random experiment is called sample space. It is denoted with S.

**EX:** In tossing a coin for two times the possible outcomes are S= {TT,HT,TH,HH}.

**Event:** The outcome of a random experiment is called an event. So, every subset of a sample space S is an event.

**Mutually exclusive events:** *E*1, *E*2, ... *En* are said to be mutually exclusive if *Ei* ∩*Ej* = , for *i* ≠*j*.

A bag contains 4 red, 5 white, 6 black balls. What is the probability that 2 balls drawn are red and white ?

**Solution.** Out of 15 balls, 2 balls can be drawn in 15*C*2 ways. Out of 4 red balls 1 ball is drawn in 4*C*1 ways and out of 5 white ball, 1 ball is chosen in 5*C*1 ways. Hence the total number of favorable cases is 4*C*1 × 5*C*1. The required probability is 4C1.5C1/15C2

**Axioms of Probability:**

(1). The numerical value of probability lies between 0 and 1. (i.e) 0<P(A)<1. (2). The sum of probabilities of all sample events is unity i.e. P(S)=1.

(3). If A and B are mutually exclusive events in sample space S then P(AUB)=P(A)+P(B).

**Notations:**

1. P(A+B) or P(AUB) indicates probability of happening events A or B.

2. P(AB) or P(A B) indicates probability of happening of both the events A and B.

3. When A and B are mutually exclusive events the P(A B)=0 since (A B)=

**Addition Law of Probability:**

If A and B are any two events, then P(AUB)=P(A)+P(B)- P(A B).

**Note:** 1. If A,B,C are any three events, then

P(AUBUC) = P(A)+P(B)+P(C)- P(A B )-P(B C)-P(C A)+P(A B C).

2.P(A *B*1 ) *B*1 = P(A)-P(A B).

3. P( *A*1 *B* )= P(B)-P(A B).

**CONDITIONAL PROBABBILITY**: The conditional probability an event B, assuming that the event A has happened, is denoted by P(B/A) and defined as, P(B/A)= P(A B)/P(A), P(A) 0.

**INDEPENDENT EVENTS:** A set of events is said to be independent if the occurrence of any one of them does not depend on the occurrence or non-occurrence of the others.

In this case, P(B/A)=P(B). So, P(A B)=P(A)P(B).

**Mutual independence.** Let *C* be a collection of events. These events are said to be mutually independent if for every non-empty subset *E*1, *E*2, ..., *En*, *P*(*E*1 ∩*E*2 ∩... ∩*En*) = *P*(*E*1) *P*(*E*2) ... *P*(*En*).

**NOTE:** 1. If the events A and B are independent, the events A1 and B are also independent.

2. If the events A and B are independent, the A1and B1 are also independent.

3. From (1) and (2) it follows that when the events A and B are independent, P(AUB)=1-[P(A1)P(B1)].

**TOTAL PROBABILITY THEOREM:** If B1, B2, B3, B4 be a set of exhaustive and mutually exclusive events, and

*n*

A is another event associated with Bi, then **P(A)=**  *P*(*Bi* )*P*(*A* / *Bi* ) **.**

*i*1

**Baye’s theorem:** If B1,B2, B3, B4 be a set of exhaustive and mutually exclusive events, and A is another

*P*(*B* )*P*(*A* / *B* )

event associated with Bi, then *P*(*B* / *A*)  *i i*

*i n*

*P*(*Bi* )*P*(*A* / *Bi* ).

*i*1

where i=1,2,…,n.

**RANDOM VARIABLES:** A random variable (RV) is a function that assigns a real number X(s) to every element sS, where S is the sample space corresponding to a random experiment.

**DISCRETE RANDOM VARIABLE:** If X is a random variable (RV) which can take a finite number or countably infinite number of values, X is called a discrete RV.

**PROBABILTY FUNCTON:** If X is a discrete RV, which can take the values x1,x2,… such that P(X=xi) = pi, then pi is called the PROBABILITY FUNCTION or PROBABILITY MASS FUNCTION, provided pi (i=1,2,3,…) satisfy the following conditions:

1. pi 0, for all i, and

2. *pi*  1

*i*

The collection of pairs {xi,pi}, i=1,2,3,.. Is called the probability distribution of the RV X.

**CONTINUOUS RANDOM VARIABLE:** If X is an RV which can take all values infinite number of values in an interval, then X is called a continuous RV.

**PROBABILITY DENSITY FUNCTION:** If X is a continuous RV such that *P*{*x*  1 *dx*  *X*  *x*  1 *dx*} *f* (*x*)*dx* ,

2 2

then f(x) is called the probability density function(pdf) of X, provided f(x) satisfies the following conditions:

*x*

1. f(x) 0, for all *xRX* . *x*  *RX P*  *pj F*() 1 *X*  *x*) *f* (*x*)*dxP*(*X*  *xi* ) *F*(*xi* ) *F*(*xi*1 )

*j*

2.

*RX*

*b*

*f* (*x*)*dx*  1. 3. *P*(*a*  *X*  *b*) *f* (*x*)*dx*

*a*

**CUMULATIVE DISTRIBUTION FUNCTION (cdf):**

If X is an RV, discrete or continuous, then P(X X) is called the cumulative distribution function of X or distribution function of X and denoted as F(x).

If X is discrete, F(x) = *pj*

*j*

*x*

*P*( *X*  *x*) *f* (*x*)*dx*

If X is continuous,

**PROPERTIES OF cdf F(x):**

**1**. F(x) is non-decreasing function of x.

2. F () =0 and *F*() 1.

3. If X is a discrete RV taking values

*P*(*X*  *xi* ) *F*(*xi* ) *F*(*xi*1)

*x*1, *x*2 , *x*3.... where

*x*1 *x*2 *x*3 .... *xi* <… then

4. If X is a continuous RV, then  *d F*(*x*) *f* (*x*) , at all pints where F(x) is differentiable.

*dx*

**MATHEMATICAL EXPECTATION:**

If X is discrete RV with probability mass function P(X=xi) = pi, i=1,2,3,.. then the mathematical expectation of X or the arithmetic mean of X, denoted as E(X), is defined as E(X)= *xi pi* .

*i*

Then Variance of a discrete RV = *x*2 *p*(*x*) *xp*(*x*) = *E*(*X* 2 ) *E*(*X* )2

If X is a continuous RV the E(X) = *xf* (*x*)*dx x*2 *p*(*x*) 2 *E*(*X* 2

2

*RX*

) *E*(*X* )

where RX is the range space of

X.